Review Two Worksheet

1. Arclength & Approximation

a. Write an integral that computes the arclength of the curve $y=e^{x/2}$ between $x=0$ & $x=2$. Use Simpsons rule with $n=4$ subintervals to estimate the value of the integral

2 2.5 3 3.5 4

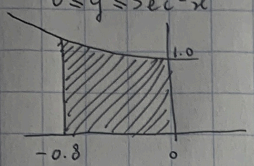
$S_n = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+f(x_n)]$

$S_n = \frac{2/4}{3}[e^1+4e^{2.5/2}+2e^{3/2}+e^{3.5/2}]$

$\boxed{S_n \approx 5.233}$

2. Center of Mass

Find the center of mass of a plate with constant density that occupies the region $-\frac{1}{4}\pi \le x \le 0$, $0 \le y \le \sec^2 x$



$C_m = \left(\frac{\int_a^b p x f(x)\,dx}{\int_a^b p\, f(x)\,dx}, \frac{\int_a^b \frac{1}{2}p(f(x))^2\,dx}{\int_a^b p f(x)\,dx}\right) = (\bar{x},\bar{y})$

Area under $\sec^2 x$ from $-1/4$ to 0

$C_m = \left(\left(\int_{-1/4}^{0} x\sec^2 x\right)/\left(\int_{-1/4}^{0}\sec^2 x\right), \left(\int_{-1/4}^{0}\frac{1}{2}\sec^4 x\,dx\right)/\left(\int_{-1/4}^{0}\sec^2 x\right)\right)$

$=\left(\frac{x\tan x - \ln(|\sec(x)|)]_{-1/4}^{0}}{\tan x]_{-1/4}^{0}}, \frac{1/2(\tan x + (\tan^3(x)/3)]_{-1/4}^{0}}{\tan x]_{-1/4}^{0}}\right)$

$\boxed{=\left(\frac{-\ln(1)-(1/4\tan 1/4 - \ln(|\sec(1/4)|))}{-\tan(-1/4)}, \frac{1/2(-(\tan -1/4 + (\tan^3(-1/4)/3)))}{-\tan(-1/4)}\right)}$

3. Net & Total Distance

You throw a ball straight up into the air with velocity 40ft/sec & catch it when it comes back down

$v(t)=40 \longrightarrow$ distance $= \left|\int 40\,dx\right| = 40x] \Rightarrow$ bounds = time = $T = \frac{2v_y}{g} = \frac{80}{9.8} = 8.16$ sec

distance $= 40x]_0^{8.16} = 40(8.16)-0$

$\boxed{=326.53 \text{ ft}}$

4. Differential Equations

Show that $y=e^{-at}\int_0^t e^{as}f(s)\,ds$ satisfies the differential equation $\frac{dy}{dt}+ay=f(t)$

$y=e^{-at}\int_0^t e^{as}f(s)\,ds$

$\frac{dy}{dt}=e^{-at}(e^{at}f(t))$

$\frac{dy}{dt} \cancel{e}^{-at2} f(t)] - f(t) = \frac{dy}{dt}+ay$

$\frac{dy}{dt}=e^{2at}\left(\frac{dy}{dt}+ay\right)$

$\frac{dy}{dt}=\frac{dy}{dt}e^{-2at}+aye^{-2at}$

$1=e^{-2at}+\frac{aye^{-2at}}{dy/dt}$

$1=e^{-2at}+e^{-2at}\cdot\frac{dy}{dt}$

$1=2e^{-2at}\frac{dy}{dt}$

$\frac{dy}{dt}+ay=f(t)$

$dy+ay=f(t)\,dt$

$dy(1+\frac{ay}{dy})=f(t)\,dt$

$\int 1+\cancel{\frac{ay}{dy}}ay\,dy=\int f(t)\,dt$

$y+\frac{ay^2}{2}=\int f(t)\,dt$

5. An electric circuit with resistance 10 ohms and inductance 2 henry's is powered by a 12 volt battery. The current I in amperes at time t (in seconds) in such a circuit satisfies the differential equation $2\frac{dI}{dt}+10I=12$

Suppose that $I=0$ when the circuit is activated at time $t=0$

① Find the current at all times $t>0$

$I=\frac{6(1-e^{-5t})}{5}$

② Find the limiting value as $t\to\infty$

As $t\to\infty$, e^{-5t} approaches ~~0~~ 0, and therefore $I=\frac{6(1-0)}{5}=\frac{6}{5}$

③ After what time is the ampere within 0.1 ampere of its limiting value?

$\int_0^{6/5} I = \int_0^{6/5}\frac{6(1-e^{-5t})}{5} = \frac{6}{5}\int_0^{6/5}(1-e^{-5t}) = \frac{6}{5}\int_0^{6/5} 1 - \frac{1}{5}e^{-5t}$

$=\frac{6}{5}(1t+\frac{1}{5}e^{-5t})]_0^{6/5}$

$\boxed{=1.2006}$